# A Study for Robustness of Objective Function and Constraints in Robust Design Optimization

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Since randomness and uncertainties of design parameters are inherent, the robust design has gained an ever increasing importance in mechanical engineering. The robustness is assessed by the measure of performance variability around mean value, which is called as standard deviation. Hence, constraints in robust optimization problem can be approached as probability constraints in reliability based optimization. Then, the FOSM (first order second moment) method or the AFOSM (advanced first order second moment) method can be used to calculate the mean values and the standard deviations of functions describing constraints and object. Among two methods, AFOSM method has some advantage over FOSM method in evaluation of probability. Nevertheless, it is difficult to obtain the mean value and the standard deviation of objective function using AFOSM method, because it requires that the mean value of function is always positive. This paper presented a special technique to overcome this weakness of AFOSM method. The mean value and the standard deviation of objective function by the proposed method are reliable as shown in examples compared with results by FOSM method.

**Key Words :** Robust Design Optimization, Reliability Based Design Optimization, Reliability Index, First Order Second Moment Method, Advanced First Order Second Moment Method

## 1. Introduction

To survive an increasingly global and competitive market, design for improving product performance and reducing cost is very important. Under this goal, there were many researches on optimal designs, but they are mainly interest in deterministic optimal design that only treats nominal values of parameters without considering uncertainty. In engineering problems, randomness and uncertainties of design parameters are inherent. Actually, applied loads and structural parameters, such as geometrical dimensions and material properties, may be subjected to random fluctuations around nominal values and thus give rise to performance variability. Therefore, randomness must be included for reasonable design.

The robust design aims to reduce the variability of mechanical performance caused by randomness of design parameters. The concept of robust design was firstly proposed by Taguchi, who use the design of experiments, and effectiveness for design improvement has been proved (Taguchi, 1987 ; Phadke, 1989). Though robust design relying on the design of experiments has been prevalent in the design of industrial products or process, it is restricted in its application as it requires planned experimentation and finite discrete variables. To overcome these limitations, several techniques for robust design were suggested, which define randomness of parameters as mean values and standard deviations. The simple method that

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calculates the mean value and the standard deviation of system response function is Monte Carlo simulation. However, since the Monte Carlo simulation needs excessive computation times, approximation methods have been developed. Ramakrishnan and Rao (1996) used second order Taylor's series approximation around mean values of random parameters to obtain mean values and the standard deviations of an objective function and constraint functions. Appling these approximations, the robust design optimization was transformed into a deterministic optimization problem. Then, the mean value of objective function was treated as a constraint. Lee and Park (2001) changed the robust design into the deterministic optimal design with multi-objective function, which was described as the mean value and the standard deviation of original objective function with robustness. In this study, the mean value and the standard deviation of a function were approximately obtained from its sensitivity at current design points and the weighting factor was introduced to solve a multi-objective function. Doltsinis and Kang (2004) also formulated robust design as a multi-objective optimal design and constraints with robustness were replaced with general constraints that add prescribed feasibility index times the standard deviations to the mean values of functions describing constraints of the robust design.

As an optimal design problem considering randomness or uncertainty, there is RBDO (Reliability Based Design Optimization). In the RBDO, probability constraints instead of constraints with robustness are defined. However, it is impossible to calculate reliability by carrying out multi-dimensional integration analytically and it is very costly to perform numerical calculation. Hence, second moment methods has been applied to calculate reliability in the RBDO. The second moment methods can be classified into two major categories such as FOSM method and AFOSM method. For the FOSM method, the mean value and the standard deviation of a function are evaluated from linearized function at the mean values of random parameters. The AFOSM method firstly calculates the reliability index that is the ratio of mean value to standard deviation. In this method, reliability index is defined as a minimum distance to a failure region (Ang and Tang, 1984). The FOSM method has been usually used to evaluate reliabilities in much literature for the RBDO, but the FOSM method may yield quite different results when failure functions with same meaning are differently described. To solve these problems, Lee and Kwak (1987-1988a; 1987b) used AFOSM method for evaluation of probability.

Even though robust design optimization is different from the RBDO in original formulation, the transformed deterministic optimization problems are the same if functions with robustness are replaced with mean values and standard deviations of functions. Under this fact, Jung and Lee (2000) treated robust design optimization as a RBDO and obtained optimal results using AFOSM method. They changed objective function of robust design into a probability constraint, but the transformed problem was different from existing robust design problem. Anyway, if the FOSM method or the AFOSM method is used to calculate a probability, the robust design optimization can be approached by the RBDO technique. To show this, evaluation of mean value and standard deviation by FOSM method and AFOSM method will be discussed and an adequate method for robust design optimization will be proposed in this paper.

## 2. Robust Design Optimization and RBDO

### 2.1 Robust design optimization

General optimization problem is written by

$$\begin{aligned} &\text{Find} \quad x \\ &\text{Minimizing} \quad J(x,z) \\ &\text{Subject to} \quad G_i(x,z) \geq 0 \\ &\qquad\qquad i=1,2,\cdots,m \end{aligned} \tag{1}$$

In above, $J(x,z)$ is an objective function and $G_i(x,z) \geq 0$ is an inequality constraint. $x$ is a controllable design variable vector and $z$ is an uncontrollable design parameter vector, which is

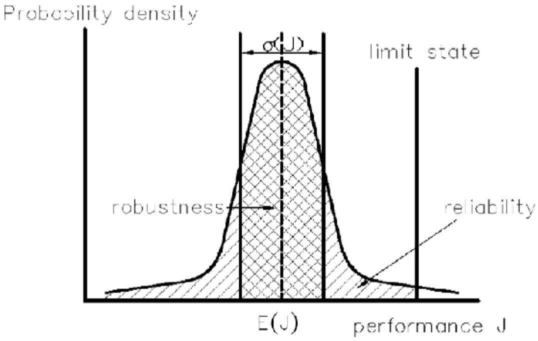


**Fig. 1** Concepts of robustness and reliability

a constant vector for the deterministic optimal design. Equality constraints must be included in the problem (1), but these are omitted for convenience.

For an optimal design with random parameters, we must consider robustness of objective function and constraints. The robustness is assessed by the measure of performance variability around mean value, most often its standard deviation, where reliability is based on the probability of failure occurrence as shown in Fig. 1. Introducing mean values and standard deviations to explain robustness, robust design optimization can be formulated as follows ;

$$\begin{aligned} &\text{Find } x \\ &\text{Minimizing } \{E(J(x,z)), \sigma(J(x,z))\} \qquad (2) \\ &\text{Subject to } E(G_i(x,z)) - \gamma_i \sigma(G_i(x,z)) \geq 0 \\ &\qquad i=1,2,\cdots,m \end{aligned}$$

Here $E(\cdot)$ and $\sigma(\cdot)$ stand for mean values and standard deviations. The quantity $\gamma_i>0$ is a prescribed feasibility index for $i$-th constraint in the robust design optimization (1). Notice that $E(x)$ and $\sigma(x)$ instead of a design variables vector $x$ must be determined. In problem (2), inequality constraints can be expressed by the following inequality constraints.

$$\frac{E(G_i(x,z))}{\sigma(G_i(x,z))} \geq \gamma_i \qquad (3)$$

### 2.2 RBDO

RBDO can generally be described as the following problem with probability constraints.

$$\begin{aligned} &\text{Find } x \\ &\text{Minimizing } \{E(J(x,z)), \sigma(J(x,z))\} \qquad (4) \\ &\text{Subject to } P_r[G_i(x,z) \leq 0] \leq p_i \\ &\qquad i=1,2,\cdots,m \end{aligned}$$

In above, $Pr[\cdot]$ means a probability and $p_i$ is an admissible failure probability. The event or state satisfying $G_i(x,z) \leq 0$ is called as a failure state. As a reference, the deterministic optimal design must satisfy constraint $G_i(x,z) \geq 0$. The RBDO is equal to robust design optimization in definition of objective function, but differs from the latter because of probability constraints.

The evaluation of probability constraint requires a reliability analysis where multiple integration for failure region $G_i(x,z) \leq 0$ is involved. To avoid this difficulty, the approximate methods such as FOSM method or AFOSM method has been developed to provide efficient solutions under assumption of Gaussian probability distributions. Then, the ratio of mean value to standard deviation has an important meaning in describing failure probability, which is particularly called as reliability index in RBDO. The reliability index $\beta_i$ is defined as

$$\beta_i = \frac{E(G_i(x,z))}{\sigma(G_i(x,z))} \qquad (5)$$

The reliability indices, the mean values and the standard deviations calculated by two methods may be different from each other for same failure functions. By the way, if the mean value and the standard deviation of $G_i(\cdot)$ are expressed as $E(G_i(x,z))$ and $\sigma(G_i(x,z))$, a probability constraint is transformed into the following inequality.

$$\frac{E(G_i(x,z))}{\sigma(G_i(x,z))} \geq \Phi^{-1}(1-p_i) \qquad (6)$$

$\Phi(\cdot)$ stands for cumulative standard Gaussian distribution function.

As a conclusion, if prescribed feasibility index $\gamma_i$ is considered as $\Phi^{-1}(1-p_i)$ that is an admissible reliability index, transformed probability constraint (6) is the same as constraint (3) of robust design optimization. Hence, it can be stated that the robust design optimization is a sort of the RBDO.

#### 2.2.1 Transformation of probability constraint by FOSM Method

FOSM method calculates the mean value and the standard deviation of a function using the linearized function that is approximately expanded by first order Taylor's series around mean value of random parameters. For convenience, a normalized random variable vector $y$ with the mean value of 0 and the standard deviation of 1 is introduced. The normalized random variable vector $y$ has following relations for design variable vector $x$ and design parameter vector $z$.

$$y_k=\frac{x_k-E(x_k)}{\sigma(x_k)} \qquad y_k=\frac{z_k-E(z_k)}{\sigma(z_k)} \tag{7}$$

The mean value and the standard deviation of a failure function for normalized random parameter vector $y$ are derived as follows;

$$E(G_i)=G_i(0) \qquad \sigma(G_i)=\sqrt{\Sigma\left(\frac{\partial G_i(0)}{\partial y_k}\right)^2} \tag{8}$$

When Eq. (6) is replaced with Eq. (8), the final RBDO problem is the same as transformed robust design optimization suggested by many researchers. If Taylor's series expansion including second order derivative is used (Ramakrishnan and Rao, 1996), mean value and standard deviation are written by

$$E(G_i)=G_i(0)+\frac{1}{2}\Sigma\left(\frac{\partial^2 G_i}{\partial y_k^2}\right)^2 \qquad \sigma(G_i)=\sqrt{\Sigma\left(\frac{\partial G_i(0)}{\partial y_k}\right)^2} \tag{9}$$

#### 2.2.2 Transformation of probability constraint by AFOSM method

Calculating the mean value and the standard deviation by FOSM, a significant error can be introduced by neglecting high order terms of nonlinear failure function. Hence, AFOSM method

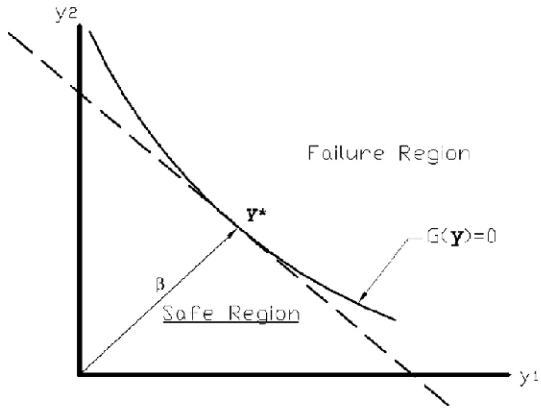


**Fig. 2** Reliability index by AFOSM method

was proposed to improve results of reliability. In the AFOSM method, reliability index is directly obtained by calculating minimum distance from origin to failure region in $y$-space, as shown in Fig. 2. This optimization problem is called as a subproblem in this paper. Using this definition, a probability constraint is transformed as follows:

$$\beta_i^* \geq \gamma_i \quad \gamma_i=\Phi^{-1}(1-p_i) \tag{10}$$

$$\beta_i^*=\underset{y\in\Omega_i}{\mathrm{Min.}}\|y\| \tag{11}$$

where

$$\Omega_i=\{y \mid G_i(y)\leq 0\}$$

Here the reliability index $\beta_i^*$ has the same meaning as the reliability index $\beta_i$ defined in previous section, even though other notation is used to distinguish the former from the latter.

After solving a subproblem (11) and getting optimal solution $y^*$, the mean value and the standard deviation of $G_i(y)$ can be evaluated from linearized function around $y^*$, The final results are as follows;

$$E(G_i(y))=-\sum_{j=1}^{r}\frac{\partial G_i}{\partial y_j}y_j^* \qquad \sigma(G_i(y))=\sqrt{\sum_{j=1}^{r}\frac{\partial G_i}{\partial y_j}y_j^*} \tag{12}$$

#### 2.2.3 Mean value and standard deviation of objective function

In case of FOSM method, the mean value and standard deviation of an objective function are simply evaluated if failure function $G_i(\cdot)$ in Eq. (8) is replaced with objective function $J(\cdot)$. Namely,

$$E(J)=J(0)$$
$$\sigma(J)=\sqrt{\sum\left(\frac{\partial J(0)}{\partial y_k}\right)^2} \tag{13}$$

It is difficult to calculate the mean value and the standard deviation of the objective function by AFOSM method, because the mean value of a function must always be larger than 0. If this condition is not satisfied, AFOSM method can not be used to calculate a reliability index. Hence, adding arbitrary constant to objective function, new function $F(\cdot)$ that has $E(F(y))>0$ are introduced.

$$F(y)=J(y)+c \tag{14}$$

The mean value and the standard deviation of a function $F(\cdot)$ have a following relation to an objective function $J(y)$.

$$E(F(y))=E(J(y))+c$$
$$\sigma(F(y))=\sigma(J(y)) \tag{15}$$

Hence, $F(\cdot)$ can be used to evaluate $E(J)$ and $\sigma(J)$. Firstly, let's consider the following subproblem (16).

$$\beta_F^*=\underset{y\in\Omega_i}{\text{Min.}}\|y\| \tag{16}$$

where

$$\Omega_i=\{y\,|\,F_i(y)\leq 0\}$$

After the solution of the above optimization is obtained, the mean value and the standard deviation of $J(y)$ can be derived from Eq. (15).

$$E(J(y))=-\sum_{j=1}^{r}\frac{\partial F_i}{\partial y_j}y_j^*+c$$
$$\sigma(J(y))=\sqrt{\sum_{j=1}^{r}\frac{\partial F_i}{\partial y_j}y_j^*} \tag{17}$$

Introducing arbitrary constant $c$, the mean value and the standard deviation of an objective function $J(\cdot)$ can be calculated by AFOSM method without difficulty. Except linear objective function of $y$, the mean value and the standard deviation of $J(\cdot)$ can be different according to the variation of $c$ values. Hence, it is very important for proposed method to determine constant $c$ well.

## 3. Comparison Between AFOSM Method and FOSM Method

If probability constraints are transformed into deterministic constraints by FOSM method or AFOSM method, it was shown that the RBDO is equivalent to the robust design optimization. However, the mean value and the standard deviation of a function obtained by each method may be different and final optimal results may not be the same. In order to find adequate method for robust design optimization, two examples for probability constraint and objective function were treated.

**〈Example 1〉** constraint $1-x_2/x_1\geq 0$ with robustness

Design variables $x_1$ and $x_2$ are random variables with Gaussian probability distribution. The mean values of these variables are positive and the standard deviations are comparatively small. In other words, $Pr[x_1\leq 0]$ and $Pr[x_2\leq 0]$ is nearly negligible. The random variables in mechanical engineering almost satisfy this condition. Then, $Pr[x_1-x_2\leq 0]$ and $Pr[1-x_2/x_1\leq 0]$ are mathematically different, but practically have same meaning in engineering view point.

Since the failure function is linear in case of $P_r[x_1-x_2\leq 0]$, reliability indiex by AFOSM method is equal to the following result that calculated by FOSM method.

$$\beta^*=\frac{E(x_1)-E(x_2)}{\sqrt{(\sigma(x_1))^2+(\sigma(x_2))^2}} \tag{18}$$

For $Pr[1-x_2/x_1\leq 0]$, the mean value and the standard deviation by FOSM method are written by

$$E(G(x))=1-\frac{E(x_2)}{E(x_1)}$$
$$\sigma(G(x))=\sqrt{\frac{(E(x_2))^2(\sigma(x_1))^2}{(E(x_1))^4}+\frac{(\sigma(x_2)^2)}{(E(x_1))^2}} \tag{19}$$

Notice that failure probabilities are generally less than 50% and they may be 1-5% in real design. Thus, $E(x_1) \geq E(x_2) > 0$ is always satisfied and the reliability index is finally derived as

$$\beta^* = \frac{E(x_1) - E(x_2)}{\sqrt{\left(\frac{E(x_2)}{E(x_1)}\sigma(x)\right)^2 + (\sigma(x_2))^2}} \quad (20)$$

As a reference, Ramakrishnan and Rao (1996) obtained the mean value and the standard deviation from second order Taylor series expansion as follows ;

$$E(G(x)) = 1 - \frac{E(x_2)}{E(x_1)} + \frac{E(x_2)}{(E(x_1))^3}$$
$$\sigma(G(x)) = \sqrt{\frac{(E(x_2))^2(\sigma(x_1))^2}{(E(x_1))^4} + \frac{(\sigma(x_2))^2}{(E(x_1))^2}} \quad (21)$$
$$\beta^* = \frac{E(x_1) - E(x_2) + \frac{E(x_2)}{(E(x_1))^3}}{\sqrt{\left(\frac{E(x_2)}{E(x_1)}\right)^2 + (\sigma(x_2))^2}}$$

On the other hand, to evaluate the mean value and the standard deviation of $1 - x_2/x_1$ by AFSOM method, subproblem (11) must be solved. At optimal solution $x^*$, the following equality must be satisfied.

$$G(x^*) = 1 - \frac{x_2^*}{x_1^*} = 0 \quad (22)$$

If Eq. (22) is expressed by normalized random variables, it means that

$$E(x_2) + y_2^* \sigma(x_2) = E(x_1) + y_1^* \sigma(x_1)$$

From this relation and Kuhn Tucker necessary condition, the optimal solutions $y_1^*$ and $y_2^*$ are calculated as

$$y_1^* \equiv -\frac{\beta^* \sigma(x_1)}{\sqrt{(\sigma(x_1))^2 + (\sigma(x_2))^2}}$$
$$y_2^* \equiv -\frac{\beta^* \sigma(x_2)}{\sqrt{(\sigma(x_1))^2 + (\sigma(x_2))^2}} \quad (23)$$

and the reliability index is obtained as follows ;

$$\beta^* = \frac{E(x_1) - E(x_2)}{\sqrt{(\sigma(x_1))^2 + (\sigma(x_2))^2}} \quad (24)$$

Finally, the mean value and the standard deviation of failure function are evaluated as

$$E(G(x)) = \frac{E(x_1) - E(x_2)}{E(x_1) + y_1^* \sigma(x_1)}$$
$$\sigma(G(x)) = \frac{\sqrt{(\sigma(x_1))^2 + (\sigma(x_2))^2}}{E(x_1) + y_1^* \sigma(x_1)} \quad (25)$$

The reliability index (24) by AFOSM method differs from reliability index (20) by FOSM method, but is the same as Eq. (18). As a conclusion, though failure functions with same meaning are differently described, the same reliability indices are always obtained by AFOSM method. Hence, constraint function in robust design optimization must be approached by AFOSM method.

〈**Example 2**〉 objective function $z_1 x_1 \sqrt{1 + x_2^2}$ with robustness

The objective function of 2-bar robust design (Ramakrishnan and Rao, 1996) is considered as an example.

$$J(x, z) = z_1 x_1 \sqrt{1 + x_2^2} \quad (26)$$

$x_1$ and $x_2$ are controllable random variables and $z_1$ with $E(z_1) = 1.0$ and $\sigma(z) = 0.5/3$ is uncontrollable random parameter. $E(x_1)$, $E(x_2)$, $\sigma(x_1)$ and $\sigma(x_2)$ are design variables in transformed robust design optimization problem. $E(x_1) = 2.05$, $E(x_2) = 1.30$, $\sigma^2(x_1) = 4.6694\text{e-}5$ and $\sigma^2(x_2) = 1.87783\text{e-}5$ were given as initial design values. Using the technique suggested in this paper, $E(J)$ and $\sigma(J)$ of the objective function (26) were calculated and compared with results by FOSM method.

If the mean value and the standard deviation of non-linear objective function are calculated by AFOSM method, the good determination of constant $c$ is very important. In order to see the variations of mean value and standard deviation for changes of $c$, four cases are selected.

Case 1 : $c = 0$
Case 2 : $c = -(E(J) - 3\sigma(J))$
Case 3 : $c = -(E(J) - 2\sigma(J))$
Case 4 : $c = -(E(J) - \sigma(J))$

In the proposed AFOSM method, $E(J)$ and $\sigma(J)$

**Table 1** Results by FOSM method and AFOSM method

| | $E(J)$ | $\sigma(J)$ | $E(J)/\sigma(J)$ |
|---|---|---|---|
| FOSM method | 3.362250e0 | 5.75805e-2 | 5.839216e1 |
| Results by Reference (1996) | 3.362252e0 | 5.75805e-2 | 5.839220e1 |
| Case 1 | 3.362759e0 | 5.60471e-2 | 5.999880e1 |
| Case 2 | 3.361818e0 | 5.72899e-2 | 5.868081e1 |
| Case 3 | 3.362055e0 | 5.73847e-2 | 5.858800e1 |
| Case 4 | 3.362201e0 | 5.74815e-2 | 5.849188e1 |

are evaluated after defining constant $c$. Hence, as it is impossible to know $E(J)$ and $\sigma(J)$ previously, the mean value and standard deviation by FOSM method are used in this example. The results for all cases are shown in Table 1.

Each case was classified according to degree that mean value $E(F)$ of new objective function $F(\cdot)$ is how far from 0. Since $\sigma(J)$ is always positive and $E(J)$ is positive, the $E(F)$ for case 1 has the largest positive value. Of course, the $E(F)$ for case 4 is near to 0. If $E(F)$ is nearly 0, $E(J)$ and $\sigma(J)$ obtained by AFOSM method converge into results by FOSM method. Except case 1 where $E(F)$ is farthest from 0, the errors of $E(J)$ and $\sigma(J)$ are about 0.013% and 0.49% compared with results by FOSM method. From these results, it can be stated that proposed method is suitable to calculate the mean value and the standard deviation of objective function.

Though arbitrary constant $c$ is added to the objective function, the mean values and the standard deviations by FOSM method are always the same. Hence, FOSM method may be better than AFOSM method. However, when FOSM method is used, different results for various functions with same meaning can be obtained. In addition to, if $c$ is well determined between $-(E(J)-\sigma(J))$ and $-(E(J)-3\sigma(J))$, the proposed AFOSM method can be applied to objective function. Under this condition, AFOSM method is recommended to evaluate the mean values and the standard deviations of all functions in robust design optimization.

## 4. Conclusion

The original form of RBDO is different from the form of robust design optimization, but transformed deterministic optimization problems are finally the same. Namely, if the mean values and the standard deviations of functions with randomness are obtained by second moment methods, a robust design optimization can be considered as a sort of RBDO. Then, it is assumed that random parameters have Gaussian probability distributions and functions are nearly linear functions for random parameters because of small standard deviation. Based on this fact, robustness can be approached as probability condition.

The FOSM method has generally been used in robust design optimization. This method may be better than AFOSM method, because it is difficult to calculate the mean value and the standard deviation of an objective function by the latter. However, if FOSM method is used to solve a robust design optimization problem, its sensitivity formula in optimization process require second order derivatives and it is not desirable. Moreover, FOSM method may yield quite different results when same state is formulated differently.

In general, the evaluation of probability by AFOSM method is relatively efficient and has some advantage over FOSM method. Hence, AFOSM method must be used to calculate the mean values and the standard deviations of all functions if possible. To achieve this goal, a special technique was presented in this paper. The mean values and the standard deviations of constraints and an objective function by AFOSM method were compared with results calculated by FOSM method. As shown in example 1, AFOSM method has superiority over FOSM method in case of probability constraints. Though the mean value and the standard deviation of an objective function were changed according to arbitrary constant $c$ that is introduced to transform the objective function $J(y)$ into the new function $F(y)$, reliable results can be obtained by the proposed method if $c$ is adequately determined. Consequently, the robust design optimization can be approached as the

RBDO and this RBDO can be solved using the proposed AFOSM method.

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